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Gravitational Field of Gyratons

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Abstract A gyraton is an object moving with the speed of light and having finite energy and internal angular momentum (spin). First, we derive the gravitational field of a gyraton in the linear approximation. After this we study solutions of the Einstein equations for gyratons. We demonstrate that these solutions in 4 and higher dimensions reduce to two linear problems in a Euclidean space. We obtain the exact solutions for relativistic gyratons, discuss their properties, and consider special examples.

1 Introduction

A gyraton is an object moving with the speed of light and having finite energy and internal angular momentum (spin). A physically interesting example of a gyraton-like object is a spinning (circular polarized) beam-pulse of the high-frequency electromagnetic or gravitational radiation. Studies of the gravitational fields of beams and pulses of light have a long history. Tolman [1] found a solution in the linear approximation. Peres [2, 3] and Bonnor [4] obtained exact solutions of the Einstein equations for a pencil of light. These solutions belong to the class of pp-waves. We discuss the generalization of these solutions to the case where the beam of radiation carries angular momentum and the number of spacetime dimensions is arbitrary [5, 6]. Such solutions are important for study mini black hole formation in a high-energy collision of two particles with spin. In the present paper we obtain general solutions for the gravitational field of gyratons and discuss their properties.

2 Gravitational Field of a Gyraton in a Linear Approximation

We consider first the gravitational field of a spinning massive point-like object in the linearized gravity. We write the linearized gravitational field in the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$,

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where $\eta_{\mu\nu}$ is the flat metric. We denote by D the total number of spacetime dimensions and by $(\bar{t}, \mathbf{x})$ the Cartesian coordinates in the Minkowski spacetime. The linearized Einstein equations has the form

$$\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{D-2}\eta_{\mu\nu}\eta^{\alpha\beta}T_{\alpha\beta}\right), \tag{1}$$

where G is the D-dimensional gravitational coupling constant. If a compact (point-like) rotating object is at rest its stress-energy components are

$$T_{\bar{t}\bar{t}} = M\delta^{D-1}(\mathbf{x}), \qquad T_{\bar{t}a} = J_{ab}\partial_b\delta^{D-1}(\mathbf{x}), \tag{2}$$

where M is the mass and J_{ab} is the angular momentum of the body. In what follows we shall consider a motion of the point-like spinning object with a constant velocity. We choose one of the spatial coordinates, $\bar{\xi}$, to be in the direction of motion, and denote the other spatial coordinates in the direction transverse to the motion by $\mathbf{x} = (x^a)$. We use notations $\bar{t}$ and $\bar{\xi}$ for the coordinates in the reference frame where the source is at rest. For simplicity we also assume that $T_{\bar{t}\bar{\xi}} = 0$.

By solving (1), one obtains the linearized metric in the form

$$ds^2 = -d\bar{t}^2 + d\bar{\xi}^2 + d\mathbf{x}^2 + 2\bar{A}_{ab}x^b dx^a d\bar{t} + \bar{\Phi}\left[d\bar{t}^2 + \frac{1}{D-3}(d\bar{\xi}^2 + d\mathbf{x}^2)\right], \tag{3}$$

$$\bar{\Phi} \sim \frac{M}{\bar{r}^{D-3}}, \qquad \bar{A}_{ab} \sim \frac{J_{ab}}{\bar{r}^{D-1}}, \tag{4}$$

$$\bar{r}^2 = \bar{\xi}^2 + r^2, \qquad r^2 = \mathbf{x}^2. \tag{5}$$

Because of the linearity, a solution for an extended one-dimensional (line-like) object oriented in $\bar{\xi}$-direction can be written in the same form (3) with the following functions $\bar{\Phi}$ and $\bar{A}_{ab}$

$$\bar{\Phi} \sim \int \frac{d\bar{\xi}'\bar{\varepsilon}(\bar{\xi}')}{[(\bar{\xi} - \bar{\xi}')^2 + \mathbf{x^2}]^{(D-3)/2}}, \tag{6}$$

$$\bar{A}_{ab} \sim \int \frac{d\bar{\xi}'\bar{j}_{ab}(\bar{\xi}')}{[(\bar{\xi} - \bar{\xi}')^2 + \mathbf{x^2}]^{(D-1)/2}}. \tag{7}$$

Here $\bar{\varepsilon}$ and $\bar{j}_{ab}$ are the mass and angular momentum densities, respectively.

To obtain a metric for a spinning source moving with the speed of light we boost the solution (3), (6), (7), that is consider this solution in a reference frame which is moving with a constant velocity. We denote by t and ξ the coordinates in the frame which is moving along $\bar{\xi}$ axis with the velocity β (in the negative direction) and $\gamma = (1-\beta^2)^{-1/2}$

$$\bar{\xi} = \gamma(\xi - \beta t) = \frac{\gamma}{\sqrt{2}}[(1-\beta)v + (1+\beta)u],$$

$$\bar{t} = \gamma(t - \beta\xi) = \frac{\gamma}{\sqrt{2}}[(1-\beta)v - (1+\beta)u].$$

Here $u=(\xi-t)/\sqrt{2}$ and $v=(\xi+t)/\sqrt{2}$ are null coordinates in the flat background spacetime. If the length l of the source in the $\bar{\xi}$-direction remains the same, its length in the moving frame because of the Lorentz contraction becomes γ^{-1}. In order to obtain an on object moving with the speed of light and having *finite duration* (length) we assume that transition to the boosted frame is accompanied by the scaling with the factor γ of the initial length of the object. We also assume that the energy, $E=\gamma M$, and the angular momentum, J_{ab}, remain fixed.

In the so-called Penrose limit, that is when $\beta \to 1$, one has

$$\bar{t} \sim -\sqrt{2}\gamma u, \qquad \bar{\xi} \sim \sqrt{2}\gamma u. \tag{8}$$

Under the above conditions one also has

$$E=\gamma \bar{M}=\int \varepsilon(u)du, \qquad \bar{\varepsilon}(\bar{\xi})=\frac{1}{\sqrt{2}\gamma^2}\varepsilon(u), \tag{9}$$

$$J_{ab}=\bar{J}_{ab}=\int j_{ab}(u)du, \qquad \bar{j}_{ab}(\bar{\xi})=\frac{1}{\sqrt{2}\gamma}j_{ab}(u). \tag{10}$$

To obtain the metric we use the following relation

$$\lim_{\gamma\to\infty}\frac{\gamma}{(\gamma^2 y^2+r^2)^{m/2}}=\frac{\sqrt{\pi}\,\Gamma((m-1)/2)}{\Gamma(m/2)}\frac{\delta(y)}{r^{m-1}}. \tag{11}$$

Using this relation one obtains

$$ds^2=-2dudv+d\mathbf{x}^2+2A_{ab}x^b dx^a du+\Phi du^2, \tag{12}$$

$$\Phi \sim \frac{\varepsilon(u)}{r^{(D-4)/2)}}, \qquad A_{ab} \sim \frac{j_{ab}(u)}{r^{(D-2)/2)}}. \tag{13}$$

For Aichelburg–Sexl metric $\varepsilon=E\delta(u)$ and $A_{ab}=0$.

3 Reduction of the Einstein Equations

Now our purpose is to obtain an exact solution of the vacuum Einstein equations (valid outside the region occupied by a gyraton) which has the asymptotic form (12–13) at large r. We assume that the metric has the same form as an asymptotic solution and write it as follows

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=-2dudv+d\mathbf{x}^2+\Phi du^2+2(\mathbf{A},d\mathbf{x})du, \tag{14}$$

$$\Phi=\Phi(u,\mathbf{x}), \qquad A_a=A_a(u,\mathbf{x}). \tag{15}$$

This metric is known as Brinkmann [7] metric. Evidently, $l^\mu\partial_\mu=\partial_v$ is the null Killing vector.

When $\Phi=\mathbf{A}=0$, the coordinates $x^1=v=(t+\xi)/\sqrt{2}$ and $x^2=u=(t-\xi)/\sqrt{2}$ are null. The coordinate u remains null for the metric (14). The metric is generated by an object moving with the velocity of light in the ξ-direction. The coordinates $(x^3,\dots,x^D)$ are coordinates of an n-dimensional space ($n=D-2$) transverse to the direction of motion.

We use bold-face symbols to denote vectors in this space. For example, $\mathbf{x}$ is a vector with components x^a $(a = 3, \dots, D)$. We denote by r the length of this vector, $r = |\mathbf{x}|$. We also denote

$$d\mathbf{x}^2 = \sum_{a=3}^{D} (dx_a)^2, \qquad (\mathbf{A}, d\mathbf{x}) = \sum_{a=3}^{D} A_a dx^a, \tag{16}$$

$$\Delta = \sum_{a=3}^{D} \partial_a^2, \qquad \operatorname{div} \mathbf{A} = \sum_{a=3}^{D} A^a_{,a}. \tag{17}$$

We assume that the sum is taken over the repeated indices and omit the summation symbol. Working in the Cartesian coordinates we shall not distinguish between upper and lower indices.

The form of the metric (14) is invariant under the following (gauge) transformation

$$v \to v + \lambda(u, \mathbf{x}), \qquad A_a \to A_a - \lambda_{,a}, \qquad \Phi \to \Phi - 2\lambda_{,u}. \tag{18}$$

It is also invariant under rescaling

$$u \to au, \qquad v \to a^{-1} v, \qquad \Phi \to a^2 \Phi, \qquad \mathbf{A} \to a\mathbf{A}. \tag{19}$$

It is easy to check that

$$l_{\mu;\nu} = 0. \tag{20}$$

It means that the null Killing vector $\mathbf{l}$ is covariantly constant. In the 4-dimensional case, space-times admitting a (covariantly) constant null vector field are called plane-fronted gravitational waves with parallel rays, or briefly pp-waves (see e.g. [8–10]). Similar terminology is often used for higher dimensional metrics (see e.g. [11, 12]).

It is possible to show that all the local scalar invariants constructed from the Riemann tensor and its covariant derivatives for the metric (14) vanish. This statement is valid *off shell*, that is the metric need not be a solution of the vacuum Einstein equations. This property is well known for 4-dimensional case, since pp-wave solutions are of Petrov type N. Generalization of this result to higher-dimensional metrics (14) with $\mathbf{A} = 0$ was given in [13, 14]. (For a general discussion of spacetimes with vanishing curvature invariants see [15–17].)

Calculations give the following non-vanishing components for the Ricci tensor

$$R_{uu} = \partial_u \operatorname{div} \mathbf{A} - \frac{1}{2}\Delta\Phi + \frac{1}{4}\mathbf{F}^2, \tag{21}$$

$$R_{au} = \frac{1}{2}\partial_b F_a{}^b, \tag{22}$$

where

$$\operatorname{div} \mathbf{A} = \partial_a A^a, \qquad \mathbf{F}^2 = F_{ab} F^{ab}, \qquad \Delta\Phi = \partial_a \partial^a \Phi. \tag{23}$$

Thus, the metric (14) is a solution of vacuum Einstein equations if and only if the following equations are satisfied

$$\partial_b F_a{}^b = 0, \tag{24}$$

$$\Delta\Phi - 2\partial_u \operatorname{div} \mathbf{A} = \frac{1}{2}\mathbf{F}^2. \tag{25}$$

4 4-Dimensional Gyratons

We consider now 4-dimensional gyratons. In this case the number of transverse dimensions is $n = 2$ and our problem reduces to 2-D electro- and magnetostatics.

Let us consider (24) for the magnetic field. Any antisymmetric tensor of the second order in a 2-dimensional space can be written as $F_{ab} = Fe_{ab}$, where e_{ab} is the totally antisymmetric tensor. Substituting this representation into (24) one obtains that $F = \text{const}$. It is easy to see that the corresponding vector potential A_a can be written as

$$A_3 = \alpha x^4, \qquad A_4 = \beta x^3, \qquad F = \beta - \alpha. \tag{26}$$

The gauge transformation (18) with $\lambda = \gamma x^3 x^4$ changes the coefficients $\alpha \to \alpha - \gamma$ and $\beta \to \beta - \gamma$ but preserves the value F.

Equation (45) takes the form

$$\Delta\psi = \frac{1}{2}F^2. \tag{27}$$

If $F \neq 0$, the solution ψ does not vanish at infinity. We exclude this case. Thus we put $F = 0$.

Let us choose a 2-dimensional contour surrounding the source at $\mathbf{x} = 0$. When $F_{ab} = 0$, the value of the integral

$$j(u) = \frac{2}{\kappa}\oint_C A_a dx^a, \qquad j(u) = \frac{1}{2}\varepsilon^{ab} j_{ab}, \tag{28}$$

does not depend on the choice of the contour C. This quantity which enters the solution (14) has the meaning of angular momentum of the gyraton. In polar coordinates (r, ϕ)

$$x^3 + ix^4 = re^{i\phi} \tag{29}$$

the corresponding potential A_a can be written as

$$A_r = 0, \qquad A_\phi = \frac{\kappa}{4\pi} j(u). \tag{30}$$

Let us consider now equation (44) for the 2-dimensional 'electric' potential φ. A solution corresponding to a point-like charge is

$$\varphi_0 = -\frac{\kappa\sqrt{2}}{2\pi}\varepsilon(u)\ln r. \tag{31}$$

Any other solution of this equation decreasing at infinity can be written as

$$\varphi = \varphi_0 + \sum_{n=-\infty}^{\infty}{}' \frac{b_n}{r^{|n|}} e^{in\phi}, \qquad \bar{b}_n = b_{-n}. \tag{32}$$

$\sum'$ indicates that the term $n = 0$ is excluded. In the electromagnetic analogy, the harmonics with $n \geq 1$ describe the field created by an electric n-pole. Since $F = 0$, $\Phi = \varphi$ and the solution for a distorted gyraton in 4-dimensional spacetime is

$$ds^2 = -2dudv + dr^2 + r^2 d\phi^2 + \frac{\kappa}{2\pi} j(u) du d\phi + \left[-\frac{\kappa\sqrt{2}}{2\pi} \varepsilon(u) \ln r + \varphi \right] du^2, \tag{33}$$

where $\varphi = \varphi(u, r, \phi)$ is given by (32) with $b_n = b_n(u)$. The quantities $\varepsilon(u)$ and $j(u)$ determine the density of the energy and angular momentum as functions of the retarded time u.

By using the gauge transformations (18) one can put $\mathbf{A} = 0$ in any simply connected region which does not contain a point $\mathbf{x} = 0$. However these transformations do not allow one to banish the potential $\mathbf{A}$ *globally*. The situation here is similar to the well known Aharonov–Bohm effect [18, 19]. The topological invariant $j(u)$ is similar to the magnetic flux in the Aharonov–Bohm case.

5 5-Dimensional Gyratons

In order to obtain a solution for the gravitational field of a 5-dimensional gyraton one needs to analyze electro- and magnetostatics in a flat 3-dimensional space.

Let us consider first the 'magnetic' equation (24). Using the standard 3-dimensional notations one can write these equations in the form

$$\mathbf{B} = \operatorname{curl} \mathbf{A}, \qquad \operatorname{curl} \mathbf{B} = 0. \tag{34}$$

The second equation implies that there exists a function Υ, the magnetic scalar potential, such that the magnetic field B is

$$\mathbf{B} = -\nabla \Upsilon. \tag{35}$$

The first of (34) implies that the magnetic potential obeys the following equation

$$\Delta \Upsilon = 0. \tag{36}$$

Let (r, θ, ϕ) be the spherical coordinates

$$x^3 + i x^4 = r \sin\theta e^{i\phi}, \qquad x^5 = r\cos\theta. \tag{37}$$

Then the general solution of (36) decreasing at infinity can be written as follows

$$\Upsilon = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} \frac{Y_{lm}(\theta, \phi)}{r^{l+1}}, \tag{38}$$

where the complex coefficients a_{lm} obey the conditions $\bar{a}_{lm} = a_{l-m}$. Here $Y_{lm}(\theta, \phi)$ are spherical harmonics

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta) e^{im\phi}. \tag{39}$$

The magnetic induction vector $\mathbf{B}$ is

$$\mathbf{B} = -\sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} \nabla \left(\frac{Y_{lm}}{r^{l+1}} \right). \tag{40}$$

The corresponding vector potential **A** is

$$\mathbf{A} = \mathbf{A}_0 - \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \frac{a_{lm}}{l} \frac{\mathbf{\Phi}_{lm}}{r^{l+1}}, \tag{41}$$

where

$$\mathbf{\Phi}_{lm}(\theta, \phi) = \mathbf{r} \times \nabla Y_{lm}(\theta, \phi). \tag{42}$$

This vector potential obeys the following gauge condition

$$\operatorname{div} \mathbf{A} = 0. \tag{43}$$

We denote by $\mathbf{A}_0$ a vector potential for $l = 0$ (the magnetic monopole) case which requires a special treatment, since in this case $\mathbf{\Phi}_0 = 0$ and ratio $\mathbf{\Phi}_{lm}/l$ is not determined.

It is convenient to write $\varPhi = \varphi + \psi$, where

$$\Delta\varphi = 0, \tag{44}$$

with a source, localized at $\mathbf{x} = 0$, and ψ is created by the "magnetic" field distribution

$$\Delta\psi = \frac{1}{2}\mathbf{F}^2. \tag{45}$$

A general solution of (44) for φ can be written as

$$\varphi = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} b_{lm} \frac{Y_{lm}(\theta, \phi)}{r^{l+1}}, \tag{46}$$

where the coefficients b_{lm} obey the conditions $\bar{b}_{lm} = b_{l-m}$. For a gyraton solution coefficients a_{lm} and b_{lm} are arbitrary functions of the retarded time u. The function ψ

$$\psi(u, \mathbf{x}) = -\frac{1}{2} \int d\mathbf{x}' \mathcal{G}_n(\mathbf{x}, \mathbf{x}') \mathbf{F}^2(u, \mathbf{x}'), \tag{47}$$

$$\mathcal{G}(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi |\mathbf{x} - \mathbf{x}'|}. \tag{48}$$

The magnetic potential $\varUpsilon$ for the magnetic monopole is $\varUpsilon = -\frac{\mu}{r}$, where μ is an arbitrary function of u. The magnetic induction vector has components

$$B_r = \frac{\mu}{r^2}, \qquad B_\theta = B_\phi = 0. \tag{49}$$

The corresponding vector potential is of the form

$$A_r = A_\theta = 0, \qquad A_\phi = -\mu \cos\theta. \tag{50}$$

The potential obeys the condition $\operatorname{div} \mathbf{A} = 0$ and the potential ψ is

$$\psi = \frac{\mu^2}{4r^2}. \tag{51}$$

The corresponding monopole solution for the gyraton is

$$ds^2 = -2dudv + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 + \left(\varphi + \frac{\mu^2(u)}{4r^2}\right)du^2 - 2\mu(u)\cos\theta dud\phi. \tag{52}$$

Here $\varphi = \varphi(u, r, \theta, \phi)$ is a solution of (44).

6 Higher Dimensional Case

Let us discuss first the scalar (electrostatic) equation (44) in the n-dimensional Euclidean space R^n

$$\Delta\varphi = 0. \tag{53}$$

To solve this equation it is convenient to decompose the potential φ into the scalar spherical harmonics [20]

$$Y^l = r^{-l}\mathcal{Y}^l, \quad \mathcal{Y}^l = C_{c_1\dots c_{l-1}} x^{c_1}\dots x^{c_{l-1}}, \tag{54}$$

where $C_{c_1\dots c_{l-1}}$ is a symmetric traceless rank-l tensor. It is easy to see that the number of linearly independent components of coefficients $C_{c_1\dots c_{l-1}}$ is

$$d_0(n,l) = \frac{(l+n-3)!(2l+n-2)}{l!(n-2)!}. \tag{55}$$

These harmonics are eigenfunctions of the invariant Laplace operator on a unit sphere S^{n-1} with eigenvalues $-l(n+l-2)$. For each l there exists $d_0(n,l)$ linearly independent harmonics. We shall use an index q to enumerate the independent harmonics. The functions Y^{lq} form a complete set, so that any smooth function F on S^{n-1} can be decomposed as

$$F = \sum_{l=0}^{\infty}\sum_q F_{lq} Y^{lq}. \tag{56}$$

Consider now a special mode $F_{lq}(r)Y^{lq}$. It is a decreasing-at-infinity solution of (53) if $F_{lq} \sim r^{-(n+l-2)}$. This can be proved by using the properties of the scalar spherical harmonics. We demonstrate this directly by using the relations (54).

First, it is easy to check that

$$\Delta\mathcal{Y}^l = 0, \qquad x^d\partial_d\mathcal{Y}^l = l\mathcal{Y}^l. \tag{57}$$

Using these relations one obtains

$$\Delta(f(r)\mathcal{Y}^l) = \left(f'' + \frac{(n+2l-1)}{r}f'\right)\mathcal{Y}^l. \tag{58}$$

Thus for $f = 1/r^{n+2l-2}$ the mode functions $f(r)\mathcal{Y}^l$ obey (53). To summarize, a general solution of the electrostatic equation (53) can be written in the form

$$\phi = \sum_{l=0}^{\infty}\sum_q \frac{\mathcal{Y}^{lq}}{r^{n+2l-2}}. \tag{59}$$

In the gyraton solution (14) $d_0(n,l)$ independent components of $C_{c_1\dots c_{l-1}}$ are arbitrary functions of u.

In a similar way, one can obtain solutions of the equations of magnetostatics in n-dimensional Euclidean space by using the vector spherical harmonics [20]. Let us denote

$$A^l_a = f(r)\mathcal{Y}^l_a, \tag{60}$$

$$\mathcal{Y}^l_a = C_{abc_1\dots c_{l-1}} x^b x^{c_1} \dots x^{c_{l-1}}. \tag{61}$$

Here $C_{abc_1\dots c_{l-1}}$ is a $(l+1)$-th-rank constant tensor which possesses the following properties: it is antisymmetric under interchange of a and b, and it is traceless under contraction of any pair of indices [20].

First, let us demonstrate that A^l_a obeys the gauge condition

$$\partial^a A^l_a = 0. \tag{62}$$

Notice that

$$\partial_a f(r) = f'(r)\frac{x^a}{r}. \tag{63}$$

Thus

$$\partial^a A^l_a = f\partial^a \mathcal{Y}^l_a = 0. \tag{64}$$

The latter equality follows from the fact that when ∂_a is acting on one of x it effectively produces a contraction of two indices in C which vanishes.

In the gauge (62) the magnetostatic field equations (24) reduce to the following equation

$$\Delta A^l_a = 0. \tag{65}$$

It is easy to get

$$\Delta \mathcal{Y}^l_a = 0, \qquad x^b \partial_b \mathcal{Y}^l_a = l\mathcal{Y}^l_a. \tag{66}$$

Using these relations one obtains

$$\Delta(f\mathcal{Y}^l_a) = \left(f'' + \frac{n+2l-1}{r} f'\right)\mathcal{Y}^l_a. \tag{67}$$

Hence $\mathcal{Y}^l_a$ is a solution of (65) if

$$f'' + \frac{n+2l-1}{r} f' = 0. \tag{68}$$

Solving this equation we get $f = 1/r^{n+2l-2}$. Hence a general decreasing at infinity solution of the magnetostatic equations in the n-dimensional space (24) can be written as

$$A_a = \sum_{l=1}^{\infty} \sum_q \frac{\mathcal{Y}^{lq}_a}{r^{n+2l-2}}. \tag{69}$$

Again, we use an index q to enumerate different linearly independent vector spherical harmonics. The total number of these harmonics for given l is [20]

$$d_1(n,l) = \frac{l(n+l-2)(n+2l-2)(n+l-3)!}{(n-3)!(l+1)!}. \tag{70}$$

In the gyraton solution (14) the coefficients $C_{abc_1\dots c_{l-1}}$ in the decomposition (69) are arbitrary functions of the retarded time u. For a given solution $\mathbf{A}$ relation (47) allows one to find ψ.

7 Summary and Discussions

We demonstrated that the vacuum Einstein equations for the gyraton-type metric (14) in an arbitrary number of spacetime dimensions D can be reduced to linear problems in the Euclidean $(D-2)$-dimensional space. These problems are: (1) To find a static electric field φ created by a point-like source; (2) To find a magnetic field $\mathbf{A}$ created by a point-like source. The retarded time u plays the role of an external parameter. One can include u-dependence by making the coefficients in the harmonic decomposition for φ and $\mathbf{A}$ to be arbitrary functions of u. After choosing the solutions of these two problems one can define ψ by means of (47). By substituting $\Phi = \varphi + \psi$ and $\mathbf{A}$ into the metric ansatz one obtains a vacuum solution of the Einstein equations.

Such a gyraton-like solution has a singularity located at the spatial point $\mathbf{x} = 0$ during some interval of the retarded time u. It means that the corresponding point-like source is moving with the velocity of light. Energy E and angular momentum J_{ab} are finite. It was demonstrated that for given energy and angular momentum the gyraton can also have other characteristics, describing the deviation of Φ from spherical symmetry (in the transverse space R^n) and the presence of higher than dipole terms in the multipole expansion of $\mathbf{A}$.

It should be emphasized that the point-like sources are certainly an idealization. In [5] it was shown that gyraton solutions can describe the gravitational field of beam-pulse spinning radiation. In such a description one uses the geometric optics approximation. For its validity the size of the cross-section of the beam must be much larger than the wave-length of the radiation. In the presence of spin J one can expect additional restrictions on the minimal size of both, the cross-section size and the duration of the pulse. As usual in physics, one must have in mind that in the possible physical applications the obtained solution is valid only outside some region surrounding the immediate neighborhood of the singularity.

As a natural generalization, it is possible to obtain gyraton solutions in the Einstein–Maxwell theory. Solutions for electrically charged gyratons were obtained in [21]. The gyraton solutions might be used for study the gravitational interaction of ultrarelativistic particles with spin (and charge). The gyraton metrics might be also interesting as possible exact solutions in the string theory. The generalization of the gyraton-type solutions to the case when a spacetime is asymptotically AdS was obtained recently in [22].

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